Given Material Properties and Dimensions

Area of strand $A_p := 3\cdot.153\text{in}^2 = 0.459\cdot\text{in}^2$

Area of comp. steel $As' := 2\cdot.2\text{in}^2 = 0.4\cdot\text{in}^2$

Comp strength concrete @ 3 days $fci' := 5\text{ksi}$

Comp strength concrete @ 28 days $fc' := 7.6\text{ksi}$

Modulus of Elasticity @ 28 days $Ec28 := 57\text{ksi}\cdot\sqrt{\frac{fc'}{psi}} = 4969\cdot\text{ksi}$

Modulus of Elasticity @ 3 days $Ec3 := 57\text{ksi}\cdot\sqrt{\frac{fci'}{psi}} = 4031\cdot\text{ksi}$

Modulus of Steel $Es := 29000\text{ksi}$

Beam Dimmensions

$i := 1..5$

Width	Height
$b_1 := 8.5\text{in}$	$h_1 := 3\text{in}$
$b_2 := 2.5\text{in}$	$h_2 := 9.5\text{in}$
$b_3 := 8\text{in}$	$h_3 := 2.5\text{in}$

Length of Beam $L := 216\text{in}$

Length of development $l_d := 12\text{in}$ *assumed*

Total Height $H := h_1 + h_2 + h_3 = 15\cdot\text{in}$

Stress in Strand

Release: $f_p := 174\text{ksi}$

Cracking: $f_{pc} := 180\text{ksi}$

Ultimate: $f_{pu} := 265\text{ksi}$

Force on Strand

Release $F_p := f_p\cdot A_p = 79.866\,\text{kip}$

Cracking: $F_{pc} := f_{pc}\cdot A_p = 82.62\cdot\text{kip}$

Ultimate: $F_{pu} := f_{pu}\cdot A_p = 121.635\,\text{kip}$

Stress of compression reinforcement $fy := 60\text{ksi}$

Transformed Section Properties at 3 Days

$$n3 := \frac{Es}{Ec3} = 7.195$$

$$A_1 := b_1 \cdot h_1 = 25.5 \cdot in^2 \qquad I_1 := \frac{b_1 \cdot (h_1)^3}{12} = 19.125 \cdot in^4 \qquad y_1 := \frac{h_1}{2} + h_2 + h_3 = 13.5 \cdot in$$

$$A_2 := b_2 \cdot h_2 = 23.75 \cdot in^2 \qquad I_2 := \frac{b_2 \cdot (h_2)^3}{12} = 178.62 \cdot in^4 \qquad y_2 := \frac{h_2}{2} + h_3 = 7.25 \cdot in$$

$$A_3 := b_3 \cdot h_3 = 20 \cdot in^2 \qquad I_3 := \frac{b_3 \cdot (h_3)^3}{12} = 10.417 \cdot in^4 \qquad y_3 := \frac{h_3}{2} = 1.25 \cdot in$$

$$A_4 := (n3 - 1) \cdot A_p \qquad I_4 := 0in^4 \qquad y_4 := \frac{h_3}{2} = 1.25 \cdot in$$

$$A_5 := (n3 - 1) \cdot As' \qquad I_5 := 0in^4 \qquad y_5 := \frac{h_1}{2} + h_2 + h_3 = 13.5 \cdot in$$

$$y_{bar} := \frac{\sum_i (A_i \cdot y_i)}{\left(\sum_i A_i\right)} = 7.757 \cdot in \qquad d_i := y_{bar} - y_i$$

$$Itr3 := \sum_i \left[I_i + A_i \cdot (d_i)^2\right] = 2104 \cdot in^4$$

$$Atr3 := \sum_i A_i = 74.572 \cdot in^2$$

$$e := (H) - y_{bar} + y_3 = 8.493 \cdot in$$

uniform SW $\omega_{sw} := 125 \frac{lbf}{ft^3} \cdot Atr3 = 64.732 \cdot plf$

Moment at ld due to SW $M_{sw} := \frac{\omega_{sw} \cdot L}{2} \cdot (l_d) - \omega_{sw} \cdot l_d \cdot \left(\frac{l_d}{2}\right) = 0.55 \cdot kip \cdot ft$

Release Stress at 3 days

Axial Stress Strand $\sigma_{pa} := \frac{-F_p}{Atr3} = -1.071 \cdot ksi$

Flexural Stress Strand $\sigma_{pf} := \frac{-(F_p \cdot e) \cdot y_{bar}}{Itr3} = -2.5 \cdot ksi$

Flex Stress DW $\sigma_{sw} := \frac{M_{sw} \cdot y_{bar}}{Itr3} = 0.024 \cdot ksi$

$f_{bot} := \sigma_{pa} + \sigma_{pf} + \sigma_{sw} = -3.5471 \cdot ksi$

$f_{top} := \sigma_{pa} - \sigma_{pf} - \sigma_{sw} = 1.405 \cdot ksi$

Check: Based on ASTM 24.5.3.1

End of simply supported members

$0.7 \cdot fci' = 3.5 \cdot ksi$

Transformed Section Properties at 28 Days

$n28 := \frac{Es}{Ec28} = 5.836$

$A_1 := b_1 \cdot h_1 = 25.5 \cdot in^2$ $I_1 := \frac{b_1 \cdot (h_1)^3}{12} = 19.125 \cdot in^4$ $y_1 := \frac{h_1}{2} + h_2 + h_3 = 13.5 \cdot in$

$A_2 := b_2 \cdot h_2 = 23.75 \cdot in^2$ $I_2 := \frac{b_2 \cdot (h_2)^3}{12} = 178.62 \cdot in^4$ $y_2 := \frac{h_2}{2} + h_3 = 7.25 \cdot in$

$A_3 := b_3 \cdot h_3 = 20 \cdot in^2$ $I_3 := \frac{b_3 \cdot (h_3)^3}{12} = 10.417 \cdot in^4$ $y_3 := \frac{h_3}{2} = 1.25 \cdot in$

$A_4 := (n28 - 1) \cdot A_p$ $I_4 := 0 in^4$ $y_4 := y_3 = 1.25 \cdot in$

$A_5 := (n28 - 1) \cdot As'$ $I_5 := 0 in^4$ $y_5 := H - 1.5in - .25in = 13.25 \cdot in$

$y_{bar} := \frac{\sum_i (A_i \cdot y_i)}{\left(\sum_i A_i\right)} = 7.763 \cdot in$ $d_i := y_{bar} - y_i$

$$Itr28 := \sum_i \left[I_i + A_i \cdot (d_i)^2 \right] = 2054 \cdot in^4$$

$$Atr28 := \sum_i A_i = 73.404 \cdot in^2$$

$$e := (H) - y_{bar} + y_3 = 8.487 \cdot in$$

Uniform SW $\omega_{sw} := 150 \frac{lbf}{ft^3} \cdot Atr28 = 76.463 \cdot plf$

Moment at L/2 due to SW $M_{sw} := \frac{\omega_{sw} \cdot L^2}{8} = 3.097 \cdot kip \cdot ft$

Cracking Load at 28 days

Axial Stress Strand $\sigma_{pa} := \frac{-F_{pc}}{Atr28} = -1.126 \cdot ksi$

Flexural Stress Strand $\sigma_{pf} := \frac{-(F_{pc} \cdot e) \cdot y_{bar}}{Itr28} = -2.65 \cdot ksi$

Flex Stress DW $\sigma_{sw} := \frac{M_{sw} \cdot y_{bar}}{Itr28} = 0.14 \cdot ksi$

Cracking Stress $\sigma_{cr} := 7.5psi \sqrt{\frac{fc'}{psi}} = 0.654 \cdot ksi$

Moment due to Live Load $M_{LL} := 1kip \cdot in$

Given

$$\left(\sigma_{pa} + \sigma_{pf} + \sigma_{sw} + \frac{M_{LL} \cdot y_{bar}}{Itr28} = \sigma_{cr} \right)$$

$$M_{LL} := Minerr(M_{LL})$$

$$Pcr := \frac{2(M_{LL})}{7.5ft} = 25.221 \cdot kip$$

Pcr: Needs to be greater than 20 kip

Ultimate Capacity at 28 days

$d := y_1 = 13.5 \cdot in$

$d' := y_4 = 1.25 \cdot in$

$b_1 = 8.5 \cdot in$

Strain of Concrete $\varepsilon c := 0.003$

$f_{pu} = 265 \cdot ksi$

$fc' = 7.6 \cdot ksi$

$$\beta_1 := \begin{cases} 0.85 & \text{if } fc' \le 4000psi \\ 0.85 - 0.05 \dfrac{(fc' - 4000psi)}{1000psi} & \text{if } 4000psi < fc' < 8000psi \\ 0.65 & \text{if } fc' \ge 8000psi \end{cases} = 0.67$$

$c := 1in$

Given

$$\left(0.85 \cdot fc' \cdot \beta_1 \cdot c \cdot b_1\right) + \min\left[As' \cdot 0.003\left(\frac{c - d'}{c}\right) \cdot Es, fy \cdot As'\right] - A_p \cdot f_{pu} = 0$$

$c := \text{Minerr}(c)$

$c = 2.785 \cdot in$

h1 has to be greater than or equal to c

$Cc := \left(0.85 \cdot fc' \cdot \beta_1 \cdot c \cdot b_1\right) = 102.455 \cdot kip$

$Cs := \left[As' \cdot 0.003\left[\dfrac{(c - d')}{c}\right] \cdot Es\right] = 19.18 \cdot kip$

$T := A_p \cdot f_{pu} = 121.635 \cdot kip$

$$Mn := \left[f_{pu} \cdot A_p \cdot \left[d - \left(\beta_1 \cdot c \cdot 0.5\right)\right] + Cs \cdot \left(\beta_1 \cdot c \cdot 0.5 - d'\right)\right]$$

$Mn = 127 \cdot kip \cdot ft$

$Pu := 39kip$ $Pn := 1kip$

$Pn := \frac{(Mn - M_{sw}) \cdot 2}{(7.5ft)} = 33.008 \cdot kip$ between 32 and 39

Shear Capacity at 28 days

$x := 1in, 2in .. 90in$

Properties of Mesh

Spacing: $S := 4in$

W4 X 4 4"X 4" (0.04in^2) $Av := 0.04in^2$

Ultimate Shear: $Vu(x) := \frac{Pn}{2} + \frac{\omega_{sw} \cdot L}{2} - \omega_{sw} \cdot (x)$

Beam properties:

Web width: $bw := b_2 = 2.5 \cdot in$

ASTM Mod Factor: [19.2.4.2] $\lambda := 0.75$

Distance from extreme compression fiber to center of prestressing strands: $dp := \max(y_5, 0.8H) = 13.25 \cdot in$

Concrete Shear Capacity:

$f_{pc}(x) := \frac{F_p}{Atr28}$

$M_{sw}(x) := \frac{\omega_{sw} \cdot L}{2} \cdot (x) - \frac{\left[\omega_{sw} \cdot (x^2)\right]}{2}$

$f_d(x) := \frac{M_{sw}(x) \cdot y_{bar}}{Itr28}$

$Mmax(x) := \frac{Pn}{2} \cdot (x)$

$f_{pe} := \frac{F_p}{Atr28} + \frac{(F_p \cdot e) \cdot y_{bar}}{Itr28}$

$Mcre(x) := \left(\frac{Itr28}{y_{bar}}\right)\left(6psi \cdot \lambda \cdot \sqrt{\frac{fc'}{psi}} + f_{pe} - f_d(x)\right)$

$Vd(x) := \frac{\omega_{sw} \cdot L}{2} - \omega_{sw} \cdot (x)$

$Vi(x) := Vu(x) - Vd(x)$

$Vci(x) := \max\left(0.6psi \cdot \lambda \cdot \sqrt{\frac{fc'}{psi}} \cdot bw \cdot dp + Vd(x) + \frac{Vi(x) \cdot Mcre(x)}{Mmax(x)}, 1.7psi \cdot \lambda \cdot \sqrt{\frac{fc'}{psi}} \cdot bw \cdot d\right)$

$$Vcw(x) := \left(3.5psi\cdot\lambda\cdot\sqrt{\frac{fc'}{psi}} + 0.3\cdot f_{pc}(x)\right)\cdot bw\cdot d + Vi(x)$$

$$Vc(x) := \min(Vci(x), Vcw(x))$$

Steel Shear Capacity: $Vs(x) := \frac{Av\cdot fy\cdot dp}{S}$

$$\phi Vn(x) := 0.75(Vc(x) + Vs(x))$$